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equation (C) *does* diverge its roots (E) from the roots (V). Therefore, *no other result being possible*, the roots that (II) has with (I) but not with ($B_{|}$) are $x_{|}=\mp_{|}2$; $y_{|}=\mp_{|}3$ and these with $x=\pm_{|}2$; $y=\pm_{|}3$ are *the eight*, and are *all* the roots of (I) (II) a fourth degree *singular* case. Q. E. D.

NOTE.—The $\pm_{|}\ldots\ldots\pm_{|}$; $\pm_{|}\ldots\ldots\mp_{|}$; or $(_{|})\ldots\ldots(_{|})$; mean "change signs in *unison only*." Thus in "SECOND" $_{|}a\ldots\ldots{}_{|}b$ (*not a* prime, etc.)

GEOMETRY.

81. Proposed by CHAS. C. CROSS, Laytonsville, Md.

A circle is drawn bisecting the lines joining the points of contact of the escribed circles with the sides produced. Another circle is drawn passing through the centers of the circles drawn tangent externally to the in-circle and internally to the sides of the triangle. Prove that the centers of these two circles, the in-center and the circumcircle are collinear.

I. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics, The Russell College, Lebanon, Va.

Let ABC be the triangle; O_a, O_b, and O_c the centers of the escribed circles tangent externally to the sides a, b, and c respectively, h and g the points of tangency of circle whose center is O_a with the sides c and b produced; e and f the points of tangency of the circle, center O_b, with the sides c and a produced; d and k the points of tangency of the circle, center O_c, with the sides b and a; O the center of the in-circle; a, b, and c the centers of the circles described tangent to circle, center O, and the sides b and c, c and a, and a and b; E, G, and K the feet of the perpendiculars from the centers a, b, and c to the side a; P, Q, and R the middle points of the lines de, gh, and hk respectively; F, L, and D the feet of the perpendiculars let fall from P, Q, and R on the side a; and O', M', M the centers of the circles through a, b, and c, A, B, and C, and P, Q, and R respectively.

Lemma—The coördinates of the center of a circle passing through (x_1, y_1), (x_2, y_2), (x_3, y_3), are given by

$$\alpha=\frac{(x_1^2+y_1^2)(y_3-y_2)+(x_2^2+y_2^2)(y_1-y_3)+(x_3^2+y_3^2)(y_2-y_1)}{2x_1(y_3-y_2)+2x_2(y_1-y_3)+2x_3(y_2-y_1)}\ldots\ldots\ldots\ldots(1).$$

$$\beta=\frac{(x_1^2+y_1^2)(x_3-x_2)+(x_2^2+y_2^2)(x_1-x_3)+(x_3^2+y_3^2)(x_2-x_1)}{2y_1(x_3-x_2)+2y_2(x_1-x_3)+2y_3(x_2-x_1)}\ldots\ldots\ldots\ldots(2).$$

$Bk=Cf=s-a$, $Cg=Ad=s-b$, $Bh=Ae=s-c$.

Taking B as origin and axes rectangular we get coördinates of M, $\{\frac{1}{2}a,\ \frac{1}{2}a\cot A\}$; of O, $\{s-b,\ r\}$; of k, $\{-[s-a],\ 0\}$; of h, $\{-[s-c]\cos B,\ -[s-c]\sin B\}$; of f, $\{s,\ 0\}$; of g, $\{a+[s-b]\cos C,\ -[s-b]\sin C\}$; of d, $\{s\cos C-a,\ s\sin C\}$; of e, $\{s\cos B,\ s\sin B\}$.

$\therefore$ of R, $\{-[\frac{1}{2}(s-a)+\frac{1}{2}(s-c)\cos B],\ -\frac{1}{2}(s-c)\sin B\}$;

of Q, $\{\frac{1}{2}[s+a+(s-b)\cos C],\ -\frac{1}{2}(s-b)\sin C\}$;

of P, $\{\frac{1}{2}[s(\cos C+\cos B)-a],\ \frac{1}{2}s(\sin C+\sin B\}$.

$BH=r\cot\frac{1}{2}B$, $OH=r$, $bE=x$, $BE=x\cot\frac{1}{2}B$.

$\therefore$ $(r-x)^2\cot^2\frac{1}{2}B+(r-x)^2=(r+x)^2$. $\therefore$ $x=r(1-\sin\frac{1}{2}B)/(1+\sin\frac{1}{2}B)$.

Let $r(1-\sin\frac{1}{2}B)/(1+\sin\frac{1}{2}B)=m$, $r(1-\sin\frac{1}{2}C)/(1+\sin\frac{1}{2}C)=n$, $r(1-\sin\frac{1}{2}A)/(1+\sin\frac{1}{2}A)=l$.

$\therefore$ coördinates of b are $(m\cot\frac{1}{2}B,\ m)$; of c, $(a-n\cot\frac{1}{2}C,\ n)$; of a, $(p\cos aBC,\ p\sin aBC)$, where $p=\sqrt{[c^2-2cl\cot\frac{1}{2}B+l^2\mathrm{cosec}^2\frac{1}{2}A]}$, and $\tan aAB=(c\tan B-l\tan B\cot\frac{1}{2}A-l)/(c-l\cot\frac{1}{2}A+l\tan B)$.

Substituting in (1) and (2) the truth of the proposition appears.

By substituting the coördinates of P, Q, R, and a, b, c in (1), (2) we get, after a prodigious amount of work, the coördinates of two points. If the line through these two points coincides with the line through O, M, the proposition is true.

[NOTE.—Dr. Zerr furnished a very beautiful figure to go with his solution, but we lacked the time to engrave it. EDITOR.]

83. Proposed by **WILLIAM HOOVER, A. M., Ph. D.,** Professor of Mathematics and Astronomy in Ohio University, Athens, O.

θ being variable, find the locus of a point whose coördinates are

$$a\tan(\theta+\alpha),\quad b\tan(\theta+\beta).$$

Solution by the PROPOSER.

The rectilinear coördinates being x and y, $x=a\tan(\theta+\alpha)$..........(1), $y=b\tan(\theta+\beta)$.............(2). (1) gives $\theta+\alpha=\tan^{-1}(x/a)$.............(3), $\theta+\beta=\tan^{-1}(y/b)$....(4). Eliminating θ, $\tan^{-1}(x/a)-\tan^{-1}(y/b)=\alpha-\beta$....(5).

Taking tangents of both members of (5) and reducing,

$$xy-\cot(\alpha-\beta)(bx-ay)+ab=0\ \ldots\ldots\ldots\ldots\ (6),$$

the equation to the required locus.

Solved in a similar manner by *COOPER D. SCHMITT*, *T. W. PALMER*, *OTTO CLAYTON*, and *G. B. M. ZERR*.

CALCULUS.

65. Proposed by **GEORGE LILLEY, Ph. D., LL. D.,** Professor of Mathematics, State University, Eugene, Ore.

A string is wound spirally 100 times around a cone 100 feet high and 2 feet in diameter at the base. Through what distance will a duck swim in unwinding the string keeping it taut at all times, the cone standing on its base and at right angles to the surface of the water ?